A Manual of

Pianoforte Tuning
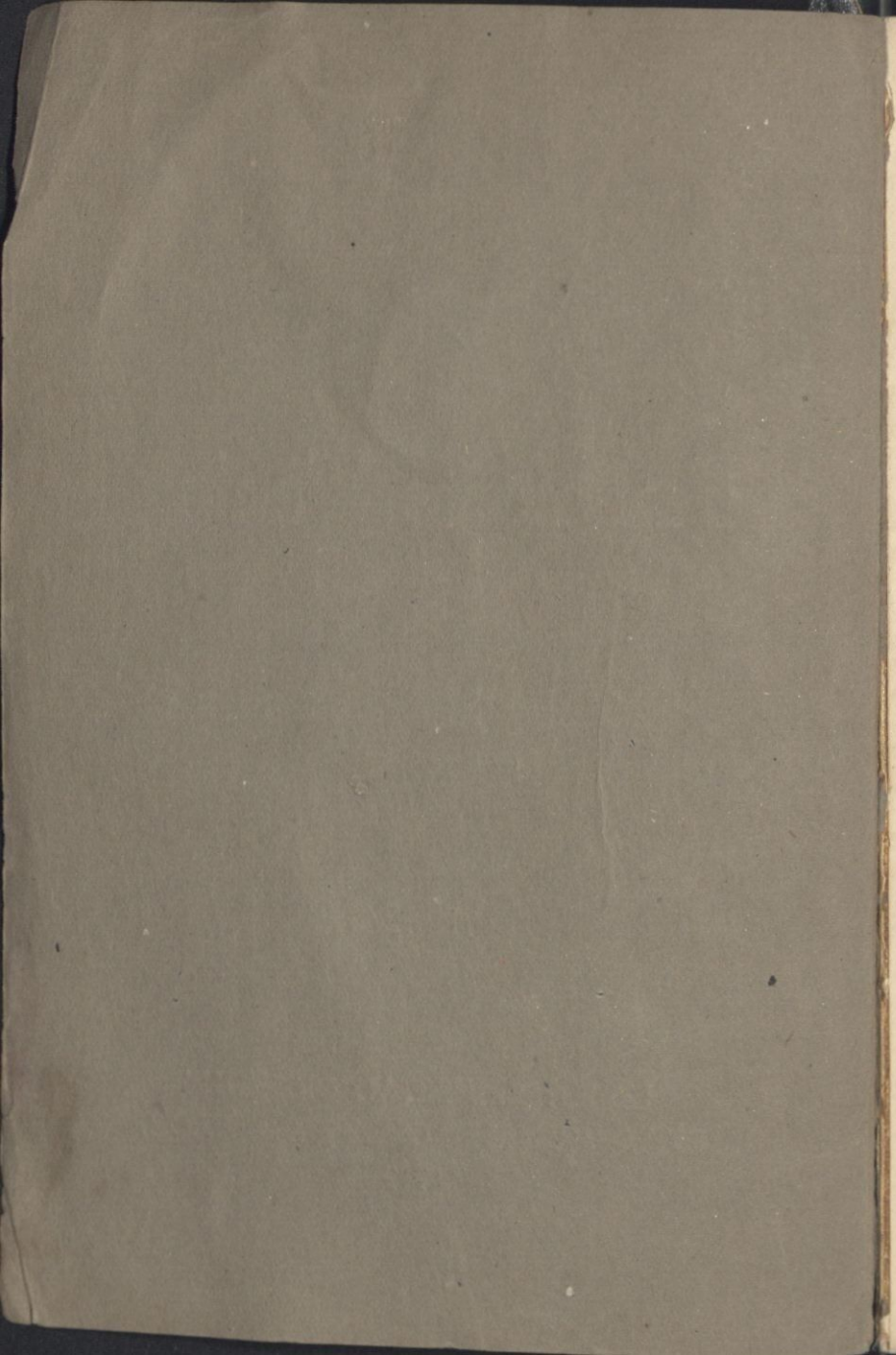
by

H. Keatley Moore, Mus.Bac., B.A.

(Of Moore & Moore)

Price 2/0

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BY

H. KEATLEY MOORE, Mus.Bac., B.A.

(of MOORE & MOORE)

Author of the Musical Articles in the National Encyclopædia; and some
time Extension Lecturer on Music for the University of London.

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A MANUAL OF
PIANOFORTE TUNING.

PART I. PRACTICAL RULES.

THIS little book is intended to be a practical manual for the use of pianoforte tuners: and therefore it is necessary to be assured that the author has a practical knowledge of his subject. His University degree shows his knowledge of the science, truly: but can he tune a pianoforte?

It must be permitted, consequently, to the author to say that he is himself a tuner, and is the son of one of that honourable fraternity and the father of another. In fact, my father, in his day, was held by most people to be the best tuner of his generation, — a claim that I should be sorry to make for myself, although I know much more about the scientific basis of tuning than my father ever did. It is, however, not the artist knowing most about the laws of colour who is the best painter. Nor is the musician knowing most about the grammar of music the best singer; indeed, the *voix de compositeur* (the wretched singing of the composer) has passed into a proverb. The scientific standard of accuracy attained is recorded in the

examination of the tuning of our house shown
on page 485 of Mr. Ellis's great English "Helm-
holtz" ("Sensations of Tone," second edition, the
fundamental authority on all matters of Musical
Tone), where No. 7 in the table of "Specimens of
Tuning in Equal Temperament" is (as stated in
the description immediately preceding) by one of
Moore & Moore's harmonium tuners, who was
trained by my father. In Art. 5 on that page is a
description of my father's division of the bear-
ings; and the result of his training seems to me
to be the best of the six specimens of tuning
done in the ordinary way. Reduced to "cents"
(where each semitone counts 100), M. & M.'s tuning
when scientifically examined proves to be as fol-
lows, taking successive semitones:—

(M. & M.)
0 98 200 298 396 498 599 702 800 898 999 1099 1199

where out of twelve notes two are exact, four are
within one hundredth of a semitone, and five
within two-hundredths, the remaining one being
four-hundredths out. Mr. Alexander J. Ellis (the
translator and editor of this immortal work) also
got Moore & Moore to tune a pianoforte for him,
but he has not printed the result. I have it, how-
ever, as measured by himself, and it is even better
than the harmonium tuning given above. It is as
follows: one right, four within one hundredth of a
semitone, six within two hundredths, and one
within three hundredths. In each case it is the
E note which is the culprit damaging these excel-
lent records.

That I may not seem to boast unduly, I append
the record of Mr. Blaikley's tuning of a harmonium,
— a tuning aided by all the resources of science,
accurately counted beats, constant blast, &c., as is
described on the same page of Helmholtz; because
it shows how impossible a thing it really is to get

the scale absolutely perfect in equal temperament. This being the best Mr. Blaikley and all his apparatus could manage, I think we ordinary unassisted tuners put up an exceedingly good show against the scientific expert. Here is Mr. Blaikley's record in cents:—

(Blaikley.)

0 100 200 300 399 499 600 700 800 900 1001 1099 1200

Please note that it is E which is the first note to "fall from virtue," even with Mr. Blaikley.

Our house of business has always paid the greatest possible attention to the training of tuners. My father had an exquisite ear; he could recognise the pitch of any note without the help of a tuning fork, and that to minute shades. "Concert pitch" to him was as real a thing as if he could actually hear and count the vibrations; and few people possess this faculty, which is called "absolute pitch." (Those who possess it are not perhaps always to be congratulated. I have a daughter who is thus gifted, and who is in a choir which specialises on unaccompanied part singing. If the pitch sinks appreciably, it means to her actual transposition; quite a laborious affair, even to a competent musician.)

Now, to a man with an extraordinary delicacy of ear like my father, the waves of tempered tuning were as palpable as the waves of the sea; and with his firm pliant wrist, it was an easy task to tighten or loosen the strings so as to make all intervals, though differing in number of waves (or beats) equal in size or distance. He did not know, nor care to know, why he did it: he was a born tuner, he knew consummately how to tune, not why. His business was to get his bearings equal, so that all keys were equally nearly in tune; and this he accomplished most remarkably well. But to us who have not "absolute pitch," a tuning fork is necessary to tell

whether a pianoforte is at concert pitch, or above it or below it. And to us who cannot hear the waves of ordinary tuning as clearly as one sees waves of the sea, rules and devices for getting an even scale are necessary. These rules I will now proceed to give, shortly, clearly, and dogmatically. The rest of the book will contain the proof of the rules; and an account of facts which underlie them, or which support and explain them.

I am not without hope that the book may be useful, perhaps interesting, to many of the general public; but I address more particularly my brother tuners.

In my father's boyhood many tuners still used a mixture of Fifths and Octaves in tuning, so as to avoid the much more difficult Fourth. It is evident that taking a C to the fork, and then tuning an octave C below, we can get a G as a Fifth up from this last; another Fifth up from this G gives us D, whence we drop to the octave D below and thence take a Fifth up to A, &c. The sequence so familiar to us all, of C, Fourth down to G, Fifth up to D, Fourth down to A, &c., is here realised by upward Fifths only, through the device of the interposed Octaves. But there is a very serious and indeed a fatal drawback. All the Fifths must be a shade flat (the reason whereof comes later in the book,—here I dogmatically say "it is so"): but all the Octaves must be perfect. The ear loses the advantage of listening to an unbroken succession of intervals all intended to be equally slightly out of tune, and of being therefore greatly assisted to achieve the desired equality of dissonance which makes good tuning. Listening to beats—or in tuner's phrase, to "waves"—is quite a different thing from listening to pure octaves, which are produced by the elimination of waves. Suppose we are climbing a dangerous stair in the dark: as soon as we have learnt the height of each stair we

can go fearlessly forward, we may even run up. But if amongst the stairs there are landings, we are thrown out; each stair becomes an unknown difficulty, to be learnt all afresh.

Therefore we have of late years been taught to use a series of bearings ranging from F to F, all in the one octave; starting from C, midway in the octave, and proceeding by Fourths down and Fifths up in the following order, each note being slightly flat to its predecessor. Tune Pitch C to the fork, then tune an octave down to Middle C. And then proceed thus:—

Middle C G D A E B F♯ C♯ G♯ (or A♭) E♭ B♭ F and down to F below.

Here G is a Fourth below middle C, D is a Fifth above G and so on: but when we reach B by a Fourth down from E, we cannot proceed in the same alternation, reaching F♯ by a Fifth up; for we must not go outside our octave of bearings, F to F, even by this one semitone. We therefore take another downward Fourth to F♯, and after that the regularity of the Fifths up and Fourths down need not be further broken.

There is also another advantage in these F to F bearings over the Fifths and Octaves method,— namely, that as the intervals are kept closely together the waves caused by their out-of-tune-ness will differ but little in number per second, whereas in wide-stretching bearings the differences are considerable; the number of waves being half as great at the octave below any note, or (which is the same thing) double as great at the octave above.

My father found that continuous bearings by Fourths and Fifths, as above, each note being slightly flat to its predecessor, often resulted in a small error being magnified, accumulating through so large a number as twelve successive progressions. He therefore invented a modification of the

orthodox bearings, by cutting them in half, and tuning the larger half as before, but the smaller half *the reverse way*. And with his delicate ear he perceived that when he reversed his path, proceeding by Fifths *down* and Fourths *up*, he had also to reverse the error (if we may call it such), so that the new note must be as much sharp of the true interval as it had been flat in the orthodox way.

My father's modified bearings, always taught in our workshops to our apprentices, were therefore as follows:—

Pitch C to Middle C, an octave down. Then, as usual at first,—namely,

Middle C G D A E B F♯ C♯ G♯,

each note a shade flat to its predecessors, falling by flattened Fourths and rising by flattened Fifths as before.

Now he started afresh for the remainder of the bearings,—thus,

Middle C down to F—B♭ E♭ A♭,

and the last note A♭ should be identical with the G♯ already tuned in the first part of the bearings.

But in this second part each note is to be tuned a little too sharp for pure harmony, and we fall by *sharpened* Fifths and rise by *sharpened* Fourths. The F is not a true Fifth below C, but slightly sharper than the truth, and the B♭ similarly is not a true Fourth above F, but slightly sharper than the truth. E♭ is a Fourth (slightly sharpened) above B♭. The next note would be, if this succession continued, A♭ a (slightly sharpened) Fifth below E♭; but in equal temperament A♭ and G♯ are the same note, and therefore the test of the accuracy of tuning is that G♯ should not need any alteration to be also nameable as A♭ in equal temperament; a Fifth below E♭, or a Fourth below

C♯, as you like. And that is all we need to complete the bearings.

You see clearly that both methods are the same. Really what is at the bottom of the whole thing is a fact of nature (which I will demonstrate later on, but as to which I here simply say, "It is so"),— namely, that 12 Fifths truly tuned one after another from the lowest A on the pianoforte up to the highest A, give a note much sharper than that produced by seven Octaves truly tuned, one after the other from the same note. If we tune our 12 true Fifths thus:—

A E B F♯ C♯ G♯ (or A♭) E♭ B♭ F C G D A

and then our 7 true Octaves thus:—

A A A A A A A, and the seventh Octave, A:—

this latter A, which is of course the true extreme treble A (since an octave admits of not the slightest variation) is exceeded quite considerably by the treble A produced from the 12 Fifths, although both the 7 Octaves and the 12 Fifths started at the same note.

You can test this for yourselves by tuning these intervals in the usual bearings-octave instead of spreading them out all over the key-board, which I may frankly tell you would be impossible. Tune then, very accurately, 12 Fifths as given above, quite smooth, real Fifths in just intonation without beats, perfectly pure: and as we pianoforte tuners are more accustomed to work from C than from A our 12 Fifths shall be the orthodox succession we all know so well:—

Middle C G D A E B F♯ C♯ G♯ (or A♭) E♭ B♭ F, and then finish with a Fifth up to Pitch C.

And if you have accomplished the difficult task of tuning accurately pure Fifths, you will find your Pitch C not truly an Octave to Middle C, but

much too sharp. It is quite evident to you that tuning a Fourth down from C to G, or down from D to A, is precisely equivalent to tuning a Fifth up from C to G, or up from D to A. All downward Fourths precisely represent upward Fifths. I suggest, in order to leave your starting point untouched, that you end with an upward Fifth to Pitch C. This Pitch C, so obtained, will be much too sharp to be an Octave to Middle C.

Now you see why all our Fifths must be smaller than the truth, for it is evident that if 12 Fifths are longer than 7 Octaves, each Fifth must be narrowed, by the same amount for each, in equal temperament, until the whole series of 12 Fifths is shortened, so as exactly to equal the 7 Octaves. But you will object that in my father's modification I direct all the intervals of the second part to be taken a trifle sharp. Yet there is no contradiction at all, if you keep your mind on the way the Fifths are reckoned. You begin with C to G: and though it suits your method to go down to G, the G you tune is really the same G, an Octave below, as the G of the upward Fifth C to G. Now the C is fixed by the fork, therefore the cutting down of the Fifth to the slightly smaller interval needed by equal temperament must all come off the G: consequently the G (whether tuned a Fourth down or a Fifth up, it makes not the slightest difference) must be a trifle flat of the truth. And so on with the next Fifth: G to D. G is now fixed, therefore D must not be a true Fifth above it, but less than a true Fifth: that is, D must be a little flat. The next Fifth is D to A, and this A must be flatter than a true Fifth; therefore, although we choose to tune A by a Fourth down from D, it must still be flatter than a true Fourth, just as if we had actually tuned it by taking a Fifth up.

Now turn to the second half of my father's.

bearings. Middle C down to F is his first interval. This is to be made smaller than a true Fifth as has been shown. But Middle C is fixed by the tuning fork : all the alteration must come, therefore, at the expense of the F, and that means that the F must be sharpened. Only by sharpening the F can you get the interval F to C less than a true Fifth, since C cannot be moved. Then he goes from F a Fifth down to B♭. It suits you in practice to go a Fourth up to B♭ instead of a Fifth down, so as to keep within your bearings, but really the note you tune as an upward Fourth represents a downward Fifth. Now since F is fixed, because you have just tuned it, the B♭ below it must come up a little, so as to make the Fifth B♭ to F less than a truly tuned Fifth,—that is B♭ must be sharpened to bring it to equal temperament : and therefore of course the B♭ you actually tuned (a Fourth above F) must also be made too sharp for truth, as it has to represent the B♭ an octave below. Whether you tune a Fifth down, or a Fourth up, in fact makes no difference : all these notes in my father's second half of the bearings are to be tuned sharp.

The thing we have to do in equal temperament is, of course, to make all our Fifths so much smaller than true Fifths that 12 of them will just fit seven Octaves. When we take our series upwards, C G A E, &c., each note as we come to it must be tuned a little flat : and of course when we take our series downwards C F B♭ A♭, &c., each note as we come to it must be tuned a little sharp.

We have been a long time getting to what after all is the practical thing in equal temperament ; namely, by *how much* these Fifths are to be made smaller so that 12 of them may be so reduced as to just equal 7 Octaves.

Presently I will show you that the total excess of the 12 Fifths is ⅛ of a semitone : and you see

therefore that each Fifth is too wide by a twelfth part of this,—namely, by a sixtieth part of a semi-tone. When you are unisoning a note the waves you at first hear are perhaps very rapid, and as you raise (or lower) the string towards a true unison the waves get slower and slower until they quite disappear as the string comes true. In scientific language these waves are always called beats, and I will later on explain why they should exist. Here I must, as before, simply say " It is so," and dogmatically assert that not only in Octaves and Unisons, but also in Fourths and Fifths, when they are not actually true, beats arise : and while a true Fifth would no more show any beats than one of your best unisons, a Fifth in equal temperament (which differs from a true Fifth by $\frac{1}{60}$ of a semitone as said above) will always beat. Another thing I leave for proof later on, but just say " It is so" for the moment, is the fact that the vibrations which really make what we call sound, increase in rapidity as we go towards the treble. If Middle C, for example, is produced by 261 vibrations per second, Pitch C, an Octave above, needs double as many—namely, 522—to produce it ; and G, between the two, needs half as many again as the lower note,—viz., a little over 391. [True G, without beats when sounded with C, needs $391\frac{1}{2}$; but equal temperament G, the G which beats with C, and which we use on the pianoforte, needs only 391.]

We find that a tuner's Fifth (an equal tempera-ment Fifth) beats on an average 6 times in 10 seconds from F up to C, and 10 times in 10 seconds from C up to G, the number increasing because of the rapidly *increasing* rate at which sound vibrates as we go up towards the treble. Now how can we measure 6 beats, or 10 beats, in 10 seconds ? Mr. Alexander Ellis shows us a very simple way ("Sensations of Tone," Second Edition, p. 486) :—

"Anyone who undertakes tuning should learn to estimate the meaning of 6, 10, 15, 20, 30 beats in 10 seconds. This is best done by short pendulums constructed of a piece of thread with one end tied to a curtain ring and the other passed through a slit in [the end of] a piece of firewood, round which it is ultimately [wound, and] tied. The stick is put under a book by the edge of a table, so that the pendulum swings freely."

Draw the thread through the slit in the firewood until there is about 9⅞ inches from the wood to the centre of the curtain ring.

"The pendulum swings backwards and forwards 120 times in 60 seconds, and hence 20 times in 10 seconds. Adjust it more accurately [by drawing the thread through the slit] by a seconds watch. Counting the swings one way only [say, the forward swings] there are 10 *double* swings in 10 seconds. [In England we always count the double swing, to *and* fro, as one swing.] By watching and counting this, say for half-an-hour [Oh, less than this would do, Mr. Ellis, even for us thick-headed tuners the tuner will learn to feel the rate."

Shorten the thread by pulling it through the slit in the firewood, until instead of 9⅞ inches it has only 4⅜: and you will have

"15 *double* swings [to *and* fro] in 10 seconds. Finally, make the length 27 1⁄16 inches...... and it will make 6 double swings in 10 seconds."

Then follows a most important caution, against a mistake that I made over and over again when counting vibrations with Mr. Ellis himself, and indeed I think even he was caught more than once.

"Remember that if you begin counting with *one* [that is, if you begin when the first swing of the pendulum *starts*, as every one naturally does], you will end with *seven* for 6, *eleven* for 10...... and so on."

For counting six, beginning when the pendulum starts, we shall have really only counted five swings, because our "sixth" is at the beginning of the sixth swing, just as our "one" was at the begin-

ning of the first swing. We must therefore always count one more in numbers than the number of swings, or seconds, or intervals of any kind that is put down to be counted. Mr. Ellis and I, when often working at this subject together, tried to begin with the word "go"; and then the 1, 2, 3, 4, 5, 6 would really represent six seconds. But we found that it was always safer and better to count six in the clumsy old way of 1, 2, 3, 4, 5, 6, 7 and then knock off one from our numbering.

But I can give a very rough approximation which is simpler even than Mr. Ellis's bit of fire-wood, and which has often served me in good stead. If you say "hickory dickory" at a moderate pace, looking at the second hand of your watch meanwhile, you will very soon be able to manage to say it exactly 10 times in 10 seconds, and just as you can always sing a song at very closely the same pace when once you have learnt it, so also you can learn the 10 seconds pace for "hickory dickory." And of course you can always check your correctness of pace when you please with your watch. Either "hickory" or "dickory" at this pace gives you half a second; and either "hick" or "dick" (being one of three syllables) gives you a sixth of a second, which is sometimes useful. Now having got your rate ("hickory dickory" ten times in ten seconds), 6 beats in 10 seconds will be a trifle quicker than one beat to each "hickory-dickory-hickory," while 10 beats in 10 seconds will be exactly one beat to each "hickory dickory." Practising with this, rough as it is, will give you the familiarity in counting very slowly so as to be able to recognise and to count 6 beats, or 10 beats, in 10 seconds : and it has the advantage of your being able to practise it at any odd moment, in a train, or out for a walk, &c.

Having studied this and got your mind well accustomed to the rates of 6 in 10 seconds and 10

in 10 seconds—and pray spend some considerable time on this before going further—then tune your Middle C (an Octave down from Pitch C by the fork) and proceed to tune G (a Fourth down) as accurately, without beats, as you can. Having got it quite smooth, flatten it till you hear 10 beats in 10 seconds when C and G are struck together. (Your firewood pendulum swinging from the top of the pianoforte after careful adjustment will help you greatly.) Then tune D a true Fifth up, without beats, and when it is quite smooth, flatten it till it gives 6 beats in 10 seconds when G and D are struck together.

But you at once object to this; "Why do you require 10 beats in 10 seconds when tuning downwards, and only 6 when tuning upwards?"

The answer is that although the actual note you tune as G is a downward Fourth from C it represents the upward Fifth from C and requires beats accordingly. Now the Fifth from Middle C up to G is much higher in the scale than the Fifth from G below Middle C up to D : and therefore it needs a higher number of beats, since the rapidity of sound-vibrations per second necessary to give the notes of our scale increases as we rise towards the treble ; and by consequence the differences between jarring vibrations must also increase in the same proportion. Suppose you have 6 beats produced when G below Middle C and D a Fifth above are struck together : then if you tune the same Fifth, G to D, an Octave above, the notes having twice the number of vibrations of the first pair, it stands to reason that you will need 12 beats instead of 6. Let C be any number, say 200: then G, if tuned quite smoothly, would be 300, and the difference between them is 100. Now take the C an octave higher (400) and G also an octave higher (600) and the difference is now 200. This is difference between

vibrations; but the same reasoning also applies to the much smaller differences between beats. Later on the nature of beats will be investigated: here we are only concerned to show that the same relative difference demands, when taken higher up the scale, a greater absolute difference than when taken lower down the scale. Therefore if we want 6 beats in the neighbourhood of Middle C, for the beats of an equally tempered Fifth, we shall want 10 beats in the neighbourhood of the G above, which is more than half way to Pitch C.

This then is the approximation, and it is extremely near to exactness, recommended by Mr. Ellis ("Sensations of Tone," Second Edition, page 489) :—

Set Pitch C to the fork, then tune a true Octave down to Middle C :—

Then down to G   ... 10 beats flat in ten seconds :
     up to D   6   ,,   ,,
   down to A   ... 10   ,,   ,,
     up to E   6   ,,   ,,
   down to B   ... 10   ,,   ,,
   down to F♯   ... 10   ,,   ,,
     up to C♯   6   ,,   ,,
   down to G♯ (A♭) 10   ,,   ,,
     up to E♭   6   ,,   ,,
   down to B♭   ... 10   ,,   ,,
     up to F   6   ,,   ,,
   down to lower F (a true Octave, no beats).

Or, as my father would prefer it, tune from Middle C as follows:—

(Flattened Fourths and Fifths.)

down to G	... 10 beats FLAT in ten sec.:
up to D	6	,,	,,
down to A	... 10 ,,	,,
up to E	6	,,	,,
down to B	... 10 ,,	,,
down to F♯	... 10 ,,	,,
up to C♯	6	,,	,,
down to G♯	... 10 ,,	,,

(Sharpened Fifths and Fourths.)

down to F	... 6 beats SHARP in ten sec.:
up to B♭	10	,,	,,
up to E♭	10	,,	,,
down to A♭	... 6 ,,	,,

and this A♭ should, without tuning, give the proper number of 6 beats sharp, and should be identical with the G♯ tuned in the previous section.

I cannot too often caution you to tune the Fourths and Fifths perfect at first, setting them so truly as to be without beats, and only then begin very gently to lower (or raise) till the proper number of beats is reached. The beats are so slow, (and moreover are not strong usually) and also so swiftly die away on the pianoforte, that it is very difficult (most tuners find it impossible) to say whether they are beats due to sharpness or to flat-

ness. With quick beats (that means when the interval is more out-of-truth) it is often fairly easy to decide.

But an experienced tuner, if he chooses, can depart from the orthodox method, given above, in the following way.

Suppose he is tuning from Middle C down to G, and the Fifth is beating 30 times in 10 seconds. His G as we know has to be only 10 beats flat, and he judges the 30 beats he hears to be 30 flat. He then very slightly raises his G, and if he counts 20 or any other lesser number after the raising, his guess was true, and he may continue with extreme gentleness and care to raise the string still further until he gets only the 10 beats required.

But if he finds that the effect of a slight raising of the G is to increase the 30 to 40 beats, he has clearly judged wrong. His 30 beats that he heard were 30 beats sharp. He must therefore lower again to 30, to 20, to 10, to unison, and then to 10 more on the flat side, which is the discrepancy required, in the direction required.

After a tuner has practised this method of tuning by beats for a considerable time he will get so accustomed to the proper rates of beating that his work will be quite easy.

It is customary for tuners to test their progress by trial chords. My father's set of trial chords was as follows. But it must be carefully remembered that none of these chords must be too smooth; all should be equally (slightly) rough :—

After tuning C G D A and E you have the chord of C :—
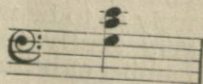
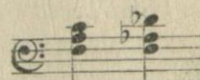Then the next note, B, gives you the chord of G :—

and the next note, F♯, gives that of D :—

and the next note, C♯, gives that of A :—
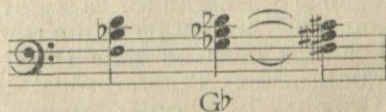
and the next note, G♯, gives that of E :—

Then, in the sharpened section, tuning F gives the chords of F and D♭ (D♭ already tuned as = C♯ and A♭ as G♯) :—

and the next note, B♭, gives those of B♭ and G♭ (F♯) (G♭ already tuned as = F♯ and D♭ as = C♯) :—

G♭

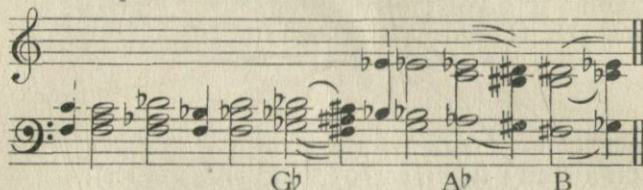and the next note, E♭, gives those of E♭ and A♭ (G♯) and B (C♭) :—

A♭   B

It may be convenient here to set out the whole bearings with the trial chords for convenience of reference.

Flattened Section.

Sharpened Section.

G♭　　　　　A♭　　　B
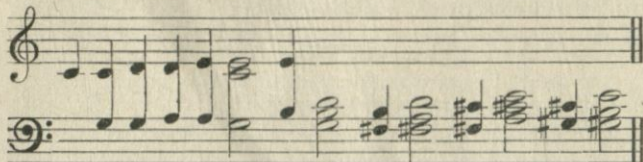
And when the bearings are complete, the chord of A♭ must be as nearly smooth (or as slightly rough, whichever you prefer to call it) as the rest. If it is not so, the bearings must be gone through again for better equalisation. But you must *never tune by chords.* Only try by chords; always tune by Fourths and Fifths. If once you fall into the sin of altering chords, you are lost. Stern bare Fourths and Fifths and Octaves are to be your only tuned intervals: and it is impossible to be too emphatic on this point.

Now, the bearings being as even as you can make them, and the Octave F tuned, it may please you to run them through in a regular sequence. This will at once show if you have accidentally favoured any key. I will put in some bass notes to mark the succession of the keys; but the upper parts may be used by themselves as soon as the bearings-octave F to F is tuned.

Sequence of Chords.

Key	C	F	B♭	E♭	A♭	D♭	G♭
Number of Flats in the Key	0	1	2	3	4	5	6

| Key	F♯	B	E	A	D	G	C
Number of Sharps in the Key	6	5	4	3	2	1	0

Hitherto the only Octaves you have tuned have been from pitch C down to Middle C and from F at the bottom of the bearings up to F at the top. We must admit at once that it is impossible to tune an Octave or a Unison quite accurately, for the simple reason that no one can tune so truly as to one beat in ten seconds, and much less than that error would be enough to spoil the perfect consonance. So what are tuners to do? The remedy is quite simple. Instead of tuning consonances, tune dissonances. The actual fork we all use is

marked, if you look at it, "522-2." That is, it is not really 522, but 520 vibrations per second (522 lessened by 2) that it sounds. The tuner sets his pitch C to beat sharp to the fork by 20 beats in 10 seconds, and that number is quite easy to count : and then his pitch C is absolutely true 522. Middle C must beat just half as often (being an octave below), and requires therefore only 10 beats in 10 seconds sharp to the pitch C fork.

But how about all the other Octaves? Is there any test by dissonance for them?

Certainly there is, and a most delicate and perfect test. Your first Octave to tune is F—F. Now the lower F beats, as you have tuned it, at the slow rate of 6 beats in 10 seconds, as against C,—that is a very little quicker than one beat in two seconds. Therefore the upper F must beat with the same C precisely twice as fast, and if you strike first
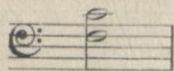
and then
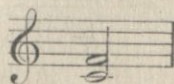
it is easy to alter the upper F till it beats with middle C just twice as fast as does the lower F.

But having got the F at the top of the bearings beating precisely 12 times in 10 seconds with middle C (for that is what it comes to), now try it with pitch C, and you will find it beats exactly at the same rate.

Thus you have a rule of dissonance enabling you to tune your Octaves perfectly.

(a) When the "halfway note" is (as in C—F—C) a Fourth above the lower note of the Octave, it should beat at the same rate with both the upper note and the lower note of the Octave: and if it does not, then the Octave note must be altered to make it do so, if it is to be a true Octave.

(*b*) When the "halfway note" is (as in F—C—F) a Fifth above the lower note of the Octave, it should beat twice as fast with the upper note as it does with the lower note: and the Octave note must be so tuned as to arrive at this result, if it is to be a true Octave.

I put the method by Fourths before that by Fifths, because you will find it much more easy. Thus I should test my two F's with B♭ rather than with C (F—B♭—F, rather than F—C—F); because both the F's would beat at the same rate with the B♭, and it is easier to recognise absolute equality than the ratio of 2 to 1. And it does not matter what may be the number of beats between the lower note of the Octave and the Fourth above it, whether 6, or 10, or 20, or 40 in ten seconds, that Fourth must have precisely the 6, or 10, or 20, or 40 beats with the upper note of the Octave. (The same thing holds good for the minor Third. C to E♭ beats precisely at the same rate as E♭ to the upper C, if the two C's are a true Octave.)

Say we want to tune the upper F of the bearings. Then we take F—B♭, which ought to beat, by the rule, 10 in 10 seconds, and make the upper F beat precisely as fast with the B♭ as does the lower F. Next we tune F♯ to F♯ in the same way: listening to the beats between F♯ and B above it and tuning the upper F♯ to beat at precisely the same rate with the B as does the lower F♯. For G to G we use C as our halfway note, for A♭ to A♭ we use D♭ for our halfway note, and so on.

[This is for upward Octaves: and common sense shows you that the method for downward Octaves must be just the converse. If you are tuning downwards from A♭ to A♭; you take as your test note the Fifth (D♭): simply because this D♭ is the Fourth to the lower note, necessary to follow the rule,—and the lower A♭ must then be brought to beat with the halfway note D♭ precisely

at the same rate as D♭ beats with the upper
A♭. For the test note in tuning G to G down-
wards, we take the downward Fifth C, and for
tuning F♯ to F♯ downwards we take as a test note
the downward Fifth B, because these are the
Fourths above the lower notes, as required by the
rule.]

PART II.

THE SCIENTIFIC BASIS OF TUNING.

Many things have been assumed in the fore-going practical part : it is now for us to investigate them.

NATURE OF MUSICAL SOUND.

Any concussion which jars the air around us makes sound : a pistol shot, or the shouts of a crowd, or the ringing of a bell, for instance. But whereas the pistol shot gives only one violent shock and the shouts of a crowd give many unequal irregular shocks, the bell vibrates for some time giving many shocks, all of them regularly occurring at equal distances one after the other. The first two are noises, the third is a musical sound. There is no other difference between noise and music.

A very large church bell vibrates slowly, and we call the effect upon the ear a low sound (bass) : the smallest bell of the peal vibrates much more quickly, and this effect upon the ear we call a high sound (treble) : but it must be observed that the few heavy bass vibrations get through

the same space of air in the same space of time as do the more numerous light treble vibrations. We might compare them to a father striding along with steps nearly a yard long and his little boy trotting by his side with steps of less than half the length, yet both arrive at the street end together.

But what is perhaps more unexpected is that a loud sound and a soft sound travel at the same speed; a gentle murmur whose vibrations scarcely move the air-particles arrives at the ear at the same time as the sound of the cannon which shakes the whole air violently, provided they both started from the same distance.

Bass and treble notes, loud and soft notes, all waves of sound move at one and the same speed, whatever the number or force of their vibrations.

Now it is evident that if we can invent apparatus to count these vibrations we shall have a splendid method of describing the pitch, or relative height of sounds. Such apparatus is to hand. Savart (1791-1841) invented one: a common form of it is a little wheel with a milled edge made to revolve in such a way that its number of rotations can be accurately counted. Let us suppose that there are 100 millings, or vertical grooves, along the edge: then if the point of a quill is brought into contact with the edge of the wheel and the wheel is rotated there will be 100 little clicks as the quill slips from one groove into the next. It is found that 522 such clicks per second (say nearly $5\frac{1}{4}$ revolutions of the wheel per second) give us the pitch C of our fork: and 553 (say just over $5\frac{1}{2}$ revolutions per second) give our D♭ in equal temperament: and 585·8 (that is 5858 in 10 seconds, or nearly $5\frac{7}{8}$ revolutions of the wheel per second) give us our D in equal temperament, and so on. Also we shall find that for the Octave above pitch C we need 1044 clicks per second (nearly $10\frac{1}{2}$ revolutions of the wheel), just double

of 522: and generally for every Octave upwards
we have to double the clicks of the notes an
Octave below.

[It may be forgiven me if I pause here
to address those who are not very well
accustomed to decimals.

Such an expression as that in the preced-
ing text, "585·8," means an amount larger
than 585 by $\frac{8}{10}$, the "decimal point" showing
where the whole number ends and the frac-
tion begins. If the expression were "585·08,"
it would mean 585 and $\frac{8}{100}$. And, generally,
one figure after the decimal point applies to
tenths, two figures apply to hundredths,
three figures to thousandths, and so on. Ex-
cuse these schoolboy reminders: to some
they may be useful.

But another matter as to the use of deci-
mals may not be so familiar. Bankers will
not pay halfpence or farthings on cheques.
If I have to pay £1 2s. 3$\frac{3}{4}$d., I must make my
cheque £1 2s. 3d. or £1 2s. 4d.; and certainly,
unless I am a mean fellow, I should make it
£1 2s. 4d. But suppose the debt is £1 2s. 2$\frac{1}{2}$d.:
that's rather a difficulty. Who's to lose the
halfpenny? I or my creditor? And in the
same way it is the habit of mathematicians
to approximate according to a certain rule,
when they are limited to a certain number
of places of decimals. Suppose all your cal-
culations have to keep within one decimal
place. What are you to do with 585·86,
with 585·55, or with 585·42?

As the first is only ·04 ($\frac{4}{100}$) short of ·90, you
call it 585·90: and that means 585·9, for
noughts have no meaning at the end of a
decimal fraction: $\frac{90}{100}$ being the same as $\frac{9}{10}$.
The third expression, 585·42, is so much

nearer to 585·4 than to 585·5, that we should call it 585·4 if limited only to one place of decimals. But what about the second expression, 585·55? It is precisely midway between 585·50 and 585·60, and we can therefore, according to circumstances, call it one or the other as suits us. The rule is then this: when the figure to be thrown out is over 5, add 1 to the preceding figure; when the figure is less than 5, the preceding figure stands unaltered; and when the figure to be thrown out is exactly 5, the preceding figure either stands as it was or has 1 added to it as circumstances seem to demand. Exactly the same rule applies if we are limited to whole numbers: thus 585·8 we should call 586 and 585·4 we should call 585. But 585·5 would be either the one or the other as seemed fittest.

You can easily see how these approximations may make calculations of vibration-numbers, &c., seem incorrect as printed: because the author calculates from the full decimal which you perhaps may not know, and not from the shortened expression which stands in the text.]

Another very famous instrument for counting sound vibrations is the siren, which is made of a perforated disc rotating above a current of wind. When one of the holes is opposite the wind-pipe, a puff of air comes out, but the solid part of the disc then closes the pipe as it passes. The next hole gives another puff, the next solid part cuts it off, and so on. If there are 10 such holes, then the siren will give 10 puffs of air for every complete rotation, and 100 for 10 rotations, and so on. A counter is affixed, to check the number of rotations very exactly. Here it is found that 522 rotations in 10 seconds (giving

in all 5220 puffs or 522 per second) make a sound which is exactly our pitch C, and 261 rotations in 10 seconds (which means 261 puffs a second) give us middle C.

Thus it is clear that clicks, or puffs of air, or indeed any regular succession of impulses, setting the air into a uniform rate of vibration, produce a note whose *pitch* corresponds with that rate of vibration. And moreover, the sound need not last for a whole second, nor for any given number of seconds, providing it vibrates at a certain uniform *rate per second*. The ear hears the proper note for that rate, although perhaps the sound has not endured for a tenth of a second. A swiftly played or sung scale of 10 notes to a second is audible quite easily, and the slightest error of the player or singer is instantly perceived, yet each note has only lasted a tenth of a second.

So much for the pitch of sounds. But their *loudness* has also to be measured; and as to this it is evident that the varying force of production is the source of the variations in volume. One who strikes a bell strikes harder if he desires a louder tone; and one who sings exerts himself more to produce a *fortissimo* than he would do for a *pianissimo* upon the same note.

The last part of musical sound to describe is its *quality*, and the origin of quality was Helmholtz's great discovery. By differences of quality we mean the difference between the sound of a flute and that of a trumpet; between the sound of a hurdy-gurdy and that of a violoncello. To get fine quality in our pianofortes we go to great expense and are at constant pains. Force we get easily enough by improving the leverage of the action; but sweetness, purity, richness, &c., which make fine quality, depend on many factors: well made hard steel strings, the most carefully constructed sound-board, a duly graduated hardness of the covering

of the hammers and the accuracy of the point at which the hammers strike the strings, &c. In fact, we tuners think much more of the tone than of the touch. A skilful player will charm his hearers, even if the touch is defective, if the tone is exquisite; and (I am sure you will wish me to add, for the glory of our calling) if the instrument is in perfect tune. Whereas, with ever so good a touch, if the tone is tinny (and especially if the tuning is also defective) the best performer can do but little for the enjoyment of his audience.

Now musical quality depends upon the relative strength of the partial-tones of which every musical tone is made up: and the beats by which we tuners tune also depend upon these very same partial-tones. It is not therefore merely a learned and philosophical enquiry upon which we enter now; it is an intensely practical matter for us tuners. I will try and make it clear, and leave out as many technicalities as I can.

If you take a string, say the C string, the lowest on the violoncello, and call the whole string "one;" and then divide it by two,—that is, stop it at the half string, and call that half "two;" and then stop it at the third of the string, and call that third "three;" and then at the quarter-string, and call the quarter "four," and so on, you will get, upon bowing these various stoppings, the following succession of sounds, which are numbered as just arranged, beginning with 1 for the whole string:—
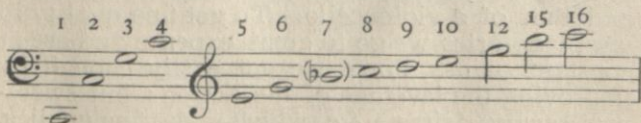
and so on to 15 16 by rapidly diminishing intervals.

All these intervals form part of our ordinary musical scale except No. 7. This No. 7 B♭ (pro-

duced by the seventh part of the string), is an unworkable note in musical composition, being very much flatter than the B♭ (the Fourth above F) which we use in actual tuning: and I also omit Nos. 11, 13 and 14; because 11 is as impossible to work with as 7 (only for the reverse reason, being much sharper than the Fourth to C, which is the F we use instead of this 11 F); and 13 is an A which is also unworkable in music; and 14 is the double of the unworkable 7. (We use 12, 15, and 16, which last is indeed the triple Octave of the keynote).

Helmholtz discovered "resonators," which are globes echoing and reinforcing certain sounds, according to their size; musical megaphones, so to speak. And after much trouble he succeeded in making a series answering to the above four-octave series of notes. It was already generally known that many sounds carried in them not only their prime tone or fundamental tone but also its Octave, and in some sounds the Fifth above the Octave was heard. But Helmholtz's resonators proved that the whole long four-octave series given above, or more or less of it, and in the order as given, was included in every musical tone. He therefore called this long series of tones "partial-tones:" and every musical sound contains them, or some of them. Perhaps there may be a few exceptional cases which only go so far as the first partial: and these are called *simple-tones*. A tuning fork, as all of us tuners know, if struck and then pressed gently on a table by its handle, flies up an octave (and some-times gives also some inharmonious upper partials, very shrill, which need not detain us, as musically they do not matter): and therefore it is evident that a tuning fork is not a simple-tone, but a tone with two partials, 1 and 2, Prime and Octave. But if a tuning-fork is gently struck, or better still

bowed with a violin-bow, and then brought near that one of Helmholtz's resonators which responds to the pitch of the fork, a simple-tone may be heard; that is the Prime-tone of the fork by itself. It is very dull and soft, and rather uninteresting: and it is found that all simple-tones are very much alike, whatever their source, and have very little distinctive character about them. As we tuners cannot be expected to possess resonators, I may suggest that a tube or bottle about $1\frac{1}{2}$ in. high or a little more, and $1\frac{1}{2}$ in. wide, with an opening of less than $\frac{1}{2}$ in., will serve as a tuning-fork resonator. It can be "tuned" by pouring in a little water, or oil; or putting in a little wax and warming it till it spreads evenly over the bottom, which last gives of course greater permanency to the "tuning" than either of the fluids can do. Such a resonator can be "tuned" to the fork (pitch C = 522), and will produce a pure simple-tone.

Investigating many ordinary musical sounds with his resonators, Helmholtz found that with care, as above described,

Tuning forks and very wide stopped organ pipes, and blowing across the open necks of wide-mouthed bottles, &c., yielded simple-tones,—i.e., first partials or prime tones only.

Pianofortes, when the strings are plucked, yield 13 partials or even more; but by striking them at about the seventh of the string, the undesirable seventh partial disappears and 1 2 3 4 5 6 (all of which are in the major common chord of the prime tone) are well brought out, and the tone becomes full and rich. Nearer the treble it is better to strike closer to the bridge, as the effect of this is to bring out the upper partials more strongly, which gives brightness to the tone, if making it at the same time rather thinner.

Violins also give six partial tones (1 2 3 4
5 6), and in skilled hands and in the lower
notes even as many as ten.

Wide Stopped Organ Pipes give only two
partials, 1 2 (Prime and Octave)

Narrow Stopped Organ Pipes give only the
uneven partials (1 3 5).

Wide Open Organ Pipes give three and
sometimes four (1 2 3 4).

Narrow Open Organ Pipes give five and
sometimes six (1 2 3 4 5 6).

The Human Voice gives 1 2 3 4 5 6 and
often also 7 and 8.

It is therefore evident that fine musical tone
is really not one simple tone but a long chord.

Examining these tones with his resonators,
Helmholtz found that in what we should, most of
us, call the finer and richer qualities of tone, the
partial tones diminished in power with some regu-
larity as their pitch increased in height, and that
the high upper partials (say beyond 8) were absent.
Of such are pianofortes, voices, open organ pipes.
Of duller quality but very sweet are flutes, wide
organ pipes, &c., with only a few partials. Where
there are high partials the tone becomes piercing :
as in the violin, if forced ; or in reed pipes, as the
oboe, &c.; or in the harmonium; or in very loud
soprano singing. And if the higher partials are
louder, relatively to the Prime-tone, than in the
musical tones already mentioned, then we get
harsh strident tones, such as the blare of the
trumpet, &c.

THE NATURE OF SOUND WAVES.

As the pianoforte has as many as (say) 6 partial-tones, these would be (by our diagram), for the note C :—

C C G C E G.

It seems desirable now to adopt a lettering which will show the pitch better than these capital letters can do ; and will henceforward avoid lengthily describing Middle C, Pitch C, and so on. This is quite easily arranged, as thus :—

Call the lowest A and B—AAA BBB.

Call the lowest C (which follows)—CC, and follow on with DD EE, &c.

Then the next octave would be—C D E F, &c. and the next (lying in the bass stave)—*c d e f,* &c.
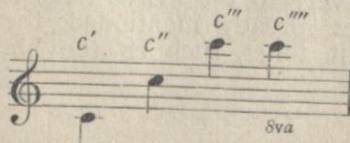
The next octave begins on Middle C—*c' d' e' f',* &c.; and the next on Pitch C—*c" d" e" f",* &c.

The next on C above the treble stave, and is thrice dashed—c''' d''' e''' f''', &c.

and the top part-octave is four times dashed—c'''' d'''' e'''' f'''' g'''' and a''''.

A seven-octave pianoforte runs, according to this nomenclature, which we shall use henceforth, from AAA in the bass to a'''' (four-dashed a) in the treble. Pitch C is c'' and Middle C is c'.

Now what, after all, are these vibrations or sound-waves which we have been so long speaking about, without hitherto attempting to describe them.

They are throbs or pulses of the air, caused by the impulses of a sounding body. A bell or a pianoforte wire, for instance, when struck, vibrate or swing, at a certain definite rate per second, according to their pitch. Pitch c'' swings to and fro, or backwards and forwards, if you prefer that description, 522 times in every second when struck by the hammer. These throbs push the particles of air adjacent to the sounding body, and push them (in the case of Pitch c'') 522 times a second, and the adjacent particles of air push those next to them at the same rate, and these last in their turn push those beyond, and so on. The whole air in every direction is with exceeding quickness made to vibrate in this manner 522 times a second: above, below, and especially in front of, and behind, and on each side of the sounding body. The particles of air adjacent to the sounding body, which have been pushed by the string (tuned to Pitch c'') travel forward under the impulse for a very short distance, transferring the push to those next them, and those in their turn passing on the push, until the impulse which is the outward part of the vibration reaches the distance of one foot, which is its vibration-distance, and reaches it in the 1044th part of a second. Then the impulse

recoils and returns to its starting point, also travel-
ling one foot in the 1044th part of a second; so that
it has been gone on the to and fro journey exactly
one 522nd of a second, and has travelled over the
space of two feet,—namely, one foot out and one
foot in.

You may reasonably think this is merely by
way of illustration : but it is the absolute fact.
For Pitch c'' needs an organ-pipe 1ft. in length,
and the sound-wave passing along the air in the
pipe has to travel the foot in length upwards, and
then the foot in length downwards, to make one
complete vibration; and, as each foot takes it one
1044th of a second to travel over, and the up-and-
down together measure two feet, the wave is done
in one 522nd of a second. In similar manner
Middle c' is produced by an organ-pipe 2ft. long:
and the up-and-down of it measures 4ft., and the
air in it vibrates only half the number of times as
for Pitch c'', namely 261. So it is with other pipes :
a pipe a third as long again as that of Pitch c''
would measure 1ft. 4in. and the vibrations would
be (to and fro, taken together) 2ft. 8in. long ; there
would be $391\frac{1}{2}$ vibrations per second (i. e., 783 in
2 seconds) and we should get the note g'. The
lengths and vibration lengths of these three pipes
would be in the ratio 6 4 3 and their vibration-
numbers per second in the ratio 2 3 4. It is quite
evident from this that in each case the pace of
sound, which must be measured from the time of
the beginning of each wave to the time of the
beginning of the next wave, is the same for all
pitches; because you see that 6×2, 4×3, and
3×4 all alike give 12. And it is the same for
every pitch, treble or bass: the higher vibration-
numbers (treble) have shorter waves but more of
them per second, the lower vibration - numbers
(bass) have longer waves but fewer of them per
second, and therefore the matter is equalised; and

all sounds move at the same pace *under the same circumstances.*

The pace of sound is, however, increased or retarded according to the medium it moves in. The more dense the medium the slower the pace of sound, and the more elastic the medium the faster the pace of sound. Sound moves faster through hydrogen gas than through air, because hydrogen is much lighter than air; and it moves slower through carbonic acid gas, because this gas is much heavier than air. It moves faster through warm air than through cold air, because warm air is more elastic than cold. You will say "Does, then, sound move faster in summer than in winter?" And I should answer "Certainly." For extremely careful measurements tell us that while sound moves at 1090 feet a second at freezing point, it moves at 1120 feet a second at 60° Fahrenheit (the temperature of a mild spring day), and, speaking roughly, the pace of sound increases about 2 feet per second for every degree warmer.

No doubt the enormous pace of light (186,000 *miles* per second) has something to do with the luminiferous ether (through which it travels) being of an elasticity so high and a density so minute that neither can be measured!

The form of the sound-wave is of such supreme importance that it may be well to try and make it clearer by a diagram.

Suppose then that we have a bell, sounding *c″*, which we know means that when struck by the clapper it will vibrate to *and* fro 522 times in a second. The stroke drives the front of the bell forward so that instead of being circular in outline it becomes slightly elliptical; and when the force of the stroke is exhausted the bell first returns to the circular form and then goes beyond that to a lessened curve, becoming elliptical in the transverse direction. That double journey completes the

wave, and the outline of the bell continues to alter
to and fro, as above, 522 times per second. Now let
a bit of this bell be represented, as thrown forward
when struck, by the curve (: and let the limit
of its return and following lessened curve be repre-
sented by) : (which last is of course a gross
exaggeration, as the outline of the bell would
still curve outward, though to a less amount than
before) : then the total distance of the swing
each way, both outward and inward, or to and
fro, is the space marked *a*. This space denotes,
therefore, the actual movement of the particles of
air, and is called the amplitude of the wave. It
is smaller for a light stroke and bigger for a
heavy stroke, and in fact it measures the force or
strength of the sound. As the sound continues it
always gets fainter, the waves being dissipated
into the surrounding air; and the space *a* becomes
smaller and smaller till it vanishes altogether on
the cessation of the sound.

b————————————————(*a*)
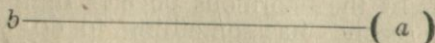
Next consider a forward movement of the bell; the
inverted-curve) advances across the space *a*,
pushing the air before it until it reaches the out-
ward-curve position (: and it is evident that from
a to *b* the air is compressed, because the impulse set
up in the space *a* travels forward towards *b* which is
supposed to be one foot away. We name this a *shell
of compression*, each particle of air squeezing for-
ward and compressing the particle in front of it.
Next consider the backward movement of the bell.
The outward-curve (moves back across the
space *a* leaving a vacuum behind it; and, as
schoolboys would say, *sucking* the air after it; and
it is evident that the air from *b* to *a* is now ex-

tended or rarified, so that we name this second part of the vibration a *shell of rarefaction*. The whole double movement of the wave will be 2 feet long and will take $\frac{1}{522}$ of a second to perform. But the actual movement to and fro of each particle of air is at the most the width of the space *a*; and it diminishes rapidly as the sound dies away. This actual air-movement also takes just the same time as the wave-movement does (namely $\frac{1}{522}$ of a second); although it is probably but the tiny fraction of an inch, while the wave is a foot long each way.

As sound in the open air, unconfined, moves in every direction, we may compare the alternating shells of compression and rarefaction to the concentric coats of an onion: and it is self-evident that each shell rapidly grows weaker, because the original impulse is spread out over a rapidly increasing surface. For the surface of a ball twice as big as another ball is not twice the original surface, but four times; and that of a ball three times as big gives nine times the surface; spherical surface increases by the square of the diameter. This is the cause why instruments and voices are so much better heard in rooms where the sound waves are prevented from dissipating into space; and indeed with speaking tubes and sound trumpets we can save up our sound to a very remarkable extent, and allow it to last out to great distances.

In a room, from the further wall, or even in the open air, from some great rock or wood, sound will be thrown back, or reflected, and will return to its source after the interval necessary to pass to and fro. This reflected sound we call an *Echo*. May I tell again the familiar tale of the Irish echo ? The sound in this echo was very perfectly reflected from a dense wood growing on an opposite hill to him who shouted and then waited two seconds. Pressed for money, the owner cut down the wood.

and found to his regret the famous echo was no
more. Of course friends arrived to hear the echo,
on the very next day; and the genial host, not to
disappoint them, took measures to make the echo
good. All went well. Every joke, or snatch of
tune, was faithfully and accurately repeated. Some
tiresome scientific person present called for silence
while he lengthily explained the cause of the echo,
and calculated the distance it had to travel, &c.
The "Echo," hearing nothing more and being
tired of waiting, electrified the company by a shout
from the distant hillside to this effect, "If ye plaze,
sorr, will Oi be for goin' home now?" And that's
that, as regards the famous Irish echo.

THE ORIGIN OF BEATS.

Our Middle c' has 261 vibrations per second, and
therefore (since all Octaves must be truly tuned),
Pitch c'' has precisely the double,—viz., 522. And
in perfectly smooth tuning (or "just intonation")
a Fifth *above* any note has half as many vibrations
again as its prime; therefore g'', a true Fifth above
c'', will have 783 vibrations per second.

But the equal-temperament g' has only 391
vibrations per second, and its Octave, g'', has of
course only the double,—viz., 782. Therefore there
will be a difference of one vibration every second,
or 10 vibrations in 10 seconds, between the equal
temperament g'' and the true g''. Let us set it out
in a diagram :—

First three partials of Middle c':	c' 261	c'' 522	g'' 783
First two partials of g' in equal temperament:		g' 391	g'' 782

Now this means that there will be one point every
second where these two g'''s will vibrate together,
and at this point therefore the sound will be
doubled. And, on the other hand, there will also
be a point in every second where a vibration of the
one note comes precisely between two vibrations
of the other, and at this point there will be silence,
supposing the two notes are of equal force. If two
men of equal strength are pulling in opposite
directions, they come to a momentary standstill;
while, if they are pulling in the same direction,
they exert double the strength of either singly.
The tone produced by our two g'''s, therefore, will
diminish, from the loud point, where the vibrations
reinforce one another, less and less, down to nothing;
and then will grow louder and louder as the vibra-
tions approach one another until they reach the
next point of union and again attain a doubled
strength. The doubled points are much more strik-
ing than the weak or silent points; and it is these
that we hear, and count as beats. We shall be able
to count 10 beats in every 10 seconds. Let us have
a diagram of some of the beats made by a note
with 300 vibrations a second against one with
200 a second, representing the vibrations of the
notes by a row of dots:—

$$1 \qquad\qquad 2 \qquad\qquad 3$$

Series of 300 . . . .

Series of 200 . . . .

$$a \qquad\qquad b \qquad\qquad c$$
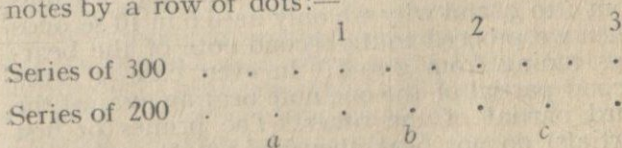
Here at 1 2 3 we hear beats, and at a b c there are
silences, our two sources of sound vibrating 9 times
in one case, against 6 times in the other. And,
speaking generally, if we want to ascertain the
number of beats between two notes not quite in
unison, when we know the vibration-numbers of
both notes per second, we subtract the one vibra-

tion-number from the other, and that gives us the number of beats per second. If we intend to count for 10 seconds, we must of course multiply the number by 10.

Pursuing the same course, starting from g, the true Fourth below Middle c' and taking its first three partials (1 the note, 2 its Octave and 3 the Fifth above), which, being effects of Nature, are always exactly in tune and then taking the d' above this g by equal temperament, and the Octave (d'') to that (which is, of course, its second partial) we get the following result :—

First three partials
  of Tenor g : g 195·55 g' 391·1 d'' 586·6,

First two partials
  of d' in equal
  temperament : d' 293 d'' 586

Here we have ·6 beats a second (and that means 6 beats in 10 seconds) between the third partial tone of g and the second of d'.

Now you see clearly why we have to listen for 10 beats in 10 seconds when we start the bearings from c' to g, and why we only need 6 in 10 seconds when we proceed to the second note of the bearings, tuning from g to d'. In every case it is the second partial of the one note beating against the third partial of the other. The primes (or first partials) do not beat, they are too far away for that ; we recognise them as Fourths or as Fifths. In order to cause beats, the two beating notes must be quite close ; in fact, we recognise them as both being *the same note*, only not truly in unison.

FURTHER EXAMINATION OF EQUAL
TEMPERAMENT.

It has been stated dogmatically in the first or practical part of this book, that equal temperament is necessary to modern music; for now-a-days musicians must have the full power to modulate into any and every key. In true tuning ("just intonation"), an instrument may be exquisitely in tune without beats, but only if it is confined to one key. If we tune C E G absolutely smooth, and then D G B (representing G B D), and then F A C, both also absolutely smooth; we shall have this one scale or key, containing C D E F G A B C, all of perfect and natural intervals. But as there was a serpent in Paradise, so in this Paradise of pure intervals, there is a traitor. The A, in the F A C section, though a true major Third to F and a true minor Third to C, is *not* a true Fifth to D. It is too sharp by the proportion of 81 to 80. A Spanish astronomer-king once said, regarding the eccentricities of the planets, that if he had been consulted at the Creation, he could have devised a much more simple movement: and in the same way, had I or any other musician created the laws of sound, I am sure we should have made A a true Fifth above D, and 12 Fifths precisely equal to 7 Octaves; and a a variety of other simplifications would have appeared. But we have to take Nature as we find her. "How, not why, is worth the guessing," as Mr. Ellis himself has said elsewhere.

Now the ratio of the vibration-numbers, comparing C with D, is as 8 to 9; but comparing D with E, it is as 9 to 10. E to F is a semitone, 15 to 16. Then while F to G is as 8 to 9 (a "major tone"), G to A is only as 9 to 10 (a "minor tone"), and yet A to B is as 8 to 9 (a "major tone"): and

B to C is a semitone, 15 to 16. Let us set them out in a diagram. Remember this is all "just intonation" or truly pure tuning : not the ordinary tuning by equal temperament which you are used to.

```
        8 - 9        8 - 9   8 - 9
     C  D  E  F  G  A  B  C
        9 - 10       9 - 10  15 - 16
```

Let us now pass to the two nearest keys.

Key of F (one flat).

```
     8 - 9        8 - 9   8 - 9
  F  G  A  B♭ C  D  E  F
     9 - 10       9 - 10  15 - 16
```

Key of G (one sharp).

```
  8 - 9        8 - 9   9 - 10
  G  A  B  C  D  E  F♯ G
     9 - 10       9 - 10  15 - 16
```

But see what has happened ! In the key of F, C to D is as 9 to 10 and D to E as 8 to 9, whereas these notes are as 8 to 9 and 9 to 10 respectively in the the key of C. And in the key of G, G to A is as 8 to 9 and A to B as 9 to 10, whereas these notes are as 9 to 10 and 8 to 9 respectively in the key of C.

In fact the D (the Sixth) in the key of F, and the A (the Second) in the key of G, have to be so seriously altered in pitch as to render these keys, when truly tuned, inharmonious with the key of C. But the evil does not end here. For *every* new flat key also has its Sixth flattened, so that B♭ (2 flats) has 2 notes flatter than the notes of the same name in the key of C ; and E♭ (3 flats) has 3 notes flatter than the same notes in the key of C ; and so forth to C♭ (7 flats), which has not a single note left which is also in tune in the key of C. And the

similar trouble comes with the sharp keys; for all
of these must have their Seconds sharpened, while
keeping the same alterations that other sharp keys
have made. Thus G (1 sharp) has 1 note sharper
than the same note in C, as we have seen; but D
(2 sharps) has 2 notes sharper than the same notes
in C; and A (3 sharps) has 3 such notes, and so on
till C♯ (7 sharps) has not a single note which was
in C.

You see now how very individual true tuning
("just intonation") is. To get one key perfect you
sacrifice all the others. At the same time, if you
want a real tuner's treat, go and listen to fine
unaccompanied part-singing, or to soft chords on
horns or trombones, or to perfect string quartets;
for these performances are truly in tune, and the
beauty of their very tones, apart from the meaning
of the music, thrills the musical listener to his
inmost soul. For over a quarter of a century I
have trained and conducted a choir for the study
of unaccompanied part-singing; in order firstly to
enjoy the pleasure of "just intonation," and
secondly to keep my ear from adopting equal
temperament as correct, seeing that I am hearing
it all day long. We must always admit that equal
temperament is a makeshift, extremely useful, and
indeed necessary, but a makeshift: "just intona-
tion" is the natural standard that the ear aims at.

How then does it come that the ear
is satisfied with this makeshift, and can
enjoy the exquisite charm of Vladimir de
Pachman, for example, playing Chopin on a
pianoforte tuned in equal temperament? The
reason lies in the construction of the ear. The
auditory nerve, with which we hear, ends in being
spread out in the inner ear underneath a mem-
brane (the basilar membrane) on the upper side of
which are living fleshy springs or arches (Corti's
arches), the span of which varies, evidently from

bass to treble, wider at the bass, narrower at the treble end of the membrane. Now there are many thousands of these Corti's arches (4500 have been counted) and if each is tuned to vibrate a certain number of times a second we could divide the whole series to give us quite 50 for every semitone in the range of musical tones audible to the human ear.

[Not all musical tones are audible : some are too high to be heard, some too low. Personally, I can hear very high notes, and I remember having acquired a headache from the shrilling of a kind of grasshopper in the early morning, while my companion, although a musician, was remarking upon the absolute stillness that prevailed. And when, at the great International Exhibition of Music at South Kensington in 1885, Mr. Alexander Ellis and I were experimenting on this subject, he could hear several more bass notes on Appun's tonometer (which is made of vibrators that gradually descend by 4 vibrations per second) than I could. There came a point at which all was silence to me, although I saw the tongues vibrating strongly. On the other hand, I beat him by a whole Octave in the treble when we were listening to a remarkably extended series of tuning forks. In both cases we found our ears had a sharply defined limit : I heard one of Appun's-tonometer vibrators and could not hear the next, Mr. Ellis heard one of the treble tuning forks and could not hear the next. There was no shading off, or partial hearing; the ear stopped dead at a certain definite vibration number.]

It is probable that not one alone but several of Corti's arches would be set vibrating by a note

at any given rate, say 522 vibrations per second,
or any other number, and if my conjecture as to
50 of them going to a semitone is correct, then 50
would serve for the vibrations between 522 and
the semitone above,—viz., 543, or (at this pitch)
about two arches to each increase by 1 in the
rapidity of vibrations. Say that 11 arches were
more or less thrown into vibration by a note
vibrating 522 times per second, then it is evident
that the middle arch would be the one most
strongly affected and would most strongly excite
the nerve fibre immediately beneath it. Those
arches on each side of it would be affected rather
less than the middle one, and so on, till those at
the outside (No. 1 and No. 11 in the little group)
would scarcely be affected at all. We should there-
fore say that the middle arch of this 11 was tuned
to our pitch c'', and the others were very near that
pitch, upwards or downwards.

This, then, is the explanation of what I may call
the "accommodation of the ear." We hear an
equally tempered Fifth; and though we know that it
is $\frac{1}{60}$ of a semitone out of tune the ear yet recognises
it as a good Fifth and is satisfied with it for every-
day working purposes; because it has actually set
in vibration (amongst other nerves) the nerves
hearing a true Fifth. Perhaps, as the point is
important we can make it clearer by a rough cal-
culation. Let the true Fifth be f'—c'', that is be-
tween notes vibrating 348 and 522 vibrations per
second. Now, in equal temperament this f' would
have to be 348·4, nearly $348\frac{1}{2}$, since the equal temp-
erament Fifth must be about $\frac{1}{60}$ of a semitone closer,
or smaller, than the true Fifth. Suppose 11 of
Corti's arches are set going by this "equal" f'. The
true f' (348) would set going a group of Corti's
arches, with 348 as the middle arch vibrating the
most strongly, thus:—

(345) $\frac{1}{2}$ 346 $\frac{1}{2}$ 347 $\frac{1}{2}$ 348 $\frac{1}{2}$ 349 $\frac{1}{2}$ 350 $\frac{1}{2}$ (351)

But the f' of equal temperament would excite a group of 11 one arch higher,—viz., from 346 to 351, with 348½ as the arch with the strongest vibration. Now the ear, like a kindly soul, admits that 348½ to 522 will do quite well as a makeshift for 348 to 522 and accepts it cheerfully, as we accept a rather old and "light" sovereign as good and lawful money.

We may now usefully draw up exact tables of "just intonation" and equal temperament side by side. By beginning at f (the f below middle c') we shall get all our bearings-octave (f to f'), and we can then draw a line and (for convenience of complete reference) proceed up as far as pitch c''. Of course we shall take Pitch c'' as 522, and Middle C as 261.

TABLES OF "JUST INTONATION" AND EQUAL TEMPERAMENT COMPARED.

	Just Intonation	Equal Temperament	Same more closely computed to three places of decimals
f	174 vibr. per sec.	174·2	(174·187)
♯	183·5	184·5	(184·537)
g	195·7	195·5	(195·546)
♯	208·8	207·15	(207·168)
a	217·5	219·5	(219·500)
♯	232	232·55	(232·550)
b	244·7	246·4	(246·420)
c'	261	261	(261·000)
♯	278·4	276·55	(276·575)
d'	293·6	293	(293·013)
♯	313·2	310·4	(310·345)
e'	326·25	328·9	(328·900)
f'	348	348·4	(348·373)

f'	$\sharp$	367	369·1	(369·074)
	g'	391·5	391	(391·092)
	$\sharp$	417·6	414·3	(414·336)
	a'	435	439	(439·000)
	$\sharp$	464	465·1	(465·100)
	b'	489·4	492·8	(492·840)
	c''	522	522	(522·000)

It must be observed that (if we consider the key above) the Third (E) is extremely sharp compared with the E of just intonation; the Fourth (F) is not very greatly sharpened, but the Sixth (A) is even worse than the Third. The Seventh (B) is worse than the Fourth.

On the other hand, the Second (D) is only a little flat, and so is the Fifth (G). Of course, the Minor Third (E♭) and the Minor Sixth (A♭) being the "inversions," which means simply the "upside downs," of the Third and Sixth (the very sharp intervals) fail in the opposite direction to them, and are extremely flat to the same notes in "just intonation."

INCOMMENSURABILITY OF (JUST) THIRDS, FIFTHS, AND OCTAVES.

If no easy relationship exists between two numbers, as for instance between 5 and 7, we say they are *incommensurable:* 5 is neither the half of 7 nor its third, nor fourth, and indeed it is not till we get to 35, into which both of them divide without remainder, that we find any close relationship between 5 and 7. They are therefore called *incommensurable*.

Turning to our diagram of partial tones, we see that the ratio of the Octave is 1 to 2, that of the Fifth 2 to 3, that of the Fourth 3 to 4, that of

the Major Third 4 to 5, and that of the Minor
Third 5 to 6. The Major Tone has the ratio 8 to 9,
the Minor Tone 9 to 10, the Semitone 15 to 16.
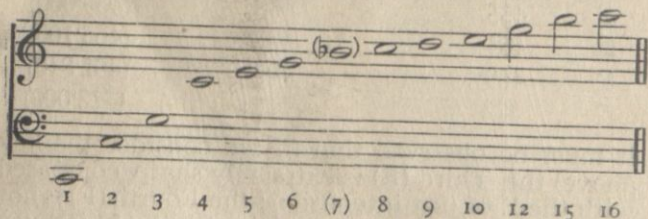
We may conveniently state the ratio 1 to 2 as $\frac{1}{2}$,
the ratio 2 to 3 as $\frac{2}{3}$, and in this fractional guise
we can add them, subtract them, &c. Only re-
member that when we add them we have to
multiply the numerators (not add them) and also
multiply, not add, the denominators. If we want,
for instance, to add a Major Third (4 : 5) to a
Minor Third (5 : 6) we must not add 4 to 5 and
5 to 6, but multiply them together, thus :—

$$4 : 5 + 5 : 6 = \frac{4}{5} \times \frac{5}{6} = \frac{20}{30}$$

But $\frac{20}{30}$ is the same as $\frac{2}{3}$: so that we here see (as
indeed the diagram shows us) that a Minor Third
(E - G) added to a Major Third (C - E) makes a
Fifth (C - G). There is a good and close relation-
ship, and it holds just as good when the Minor
Third comes first. Thus the Major Third (C - E)
added above the Minor Third (A to C) gives the
Fifth (A - E). Yes, that is so. But each kind of
Third taken by itself is completely incom-
mensurable with a Fifth.

The Major Third 4 : 5 has no evident relation-
ship with the Fifth 2 : 3, it is not a half of it, nor
of course a third of it. Nor is the Minor Third
5 : 6 any more nearly related to the Fifth. Simply
these two incommensurable halves unite to form a

symmetrical whole. Nor have either of these
Thirds any close relationship with the Octave (1 : 2).

We can also show the incommensurability by
tuning. For while 3 Major Thirds in equal tem-
perament make exactly an Octave, C-E, E-G♯, A♭-C
(A♭ being the same in equal temperament as G♯),
they fall far short of it in "just intonation." Try
it and see. Tune the 3 Major Thirds truly
without beats and you will find the Octave wildly
flat. Or you may see it in figures as clearly. Add
3 Major Thirds (4 to 5),—viz.,

$$\frac{4 \times 4 \times 4}{5 \times 5 \times 5} \text{ and you get } \frac{64}{125} \text{ (64 to 125).}$$

But the Octave (1 to 2) at the same rate gives $\frac{64}{128}$

(64 to 128). Therefore three Major Thirds are to
the Octave as 128 to 125.

So (only precisely in the other direction) with
the Minor Third. In equal temperament, four of
them make exactly an Octave, as thus :—

C—E♭, E♭—G♭, F♯ $\left(\substack{\text{the same as G2 in equal} \\ \text{temperament}}\right)$—A, A—C.

Tune them accurately without beats, and you will
find your top C wildly sharp. Or look at it in
figures. Add four Minor Thirds (5 : 6) thus :—

$$\frac{5 \times 5 \times 5 \times 5}{6 \times 6 \times 6 \times 6} \text{ and you get } \frac{625}{1296}.$$

But an Octave (1 : 2) from 625 is only 1250.
Therefore three Minor Thirds are to the Octave as
1250 to 1296 ; or, roughly, as 25 to 26.

So also with the Fifth and the Octave : they are
quite incommensurable. On our piano, by equal
temperament, 12 Fifths (AAA to a'''') precisely reach
7 Octaves (AAA to a''''). But not so if the Fifths

are accurately tuned without beats. Tune all your bearings dead smooth, and try. Only, when you reach the end (the last to be tuned having been $b\flat$ to $e\flat'$) tune the f' up from the f already set, and then go on another Fifth up to c'', as recommended on page 7, so as not to disturb Middle c', your starting point. You will find the Pitch c'' thus derived from "just" bearings very sharp to the Middle c' from which the bearings started. The exact amount is a rather formidable looking fraction,— namely, what learned men call the "Pythagorean comma," $\frac{524288}{531441}$. But we can make it more easily comprehensible by saying that it is about a fifth part of a semitone : the ratio of a semitone being 15 : 16.

You may reach the "Pythagorean comma" by putting your ("just") bearings into figures, thus :— c' down to g is the ratio of $\frac{4}{3}$, g up to d' is that of $\frac{2}{3}$; and so go on, putting $\frac{4}{3}$ for every downward Fourth and $\frac{2}{3}$ for every upward Fifth. You have therefore to add (remember that addition of ratios means multiplication) these 12 ratios :—

$$\text{Ratios} \quad \frac{4}{3} + \frac{2}{3} + \frac{4}{3} + \frac{2}{3} + \frac{4}{3} + \frac{4}{3} +$$

$$\frac{2}{3} + \frac{4}{3} + \frac{2}{3} + \frac{4}{3} + \frac{4}{3} + \frac{2}{3} = \frac{524288}{531441}$$

Or you may start from an imaginary note with one vibration a second and go up 12 Fifths from it. As the Fifth is 2 : 3 the vibration of each note is half as quick again as its predecessor. The series will therefore run thus : 1 to begin with, first Fifth 1·5 ; second, 2·25 ; third, 3·375, and so on ; and the twelfth Fifth will have the vibration number 129·746 and nine more places of decimals,—call it 129·75. Whereas $2 \times 2 \times 2$ &c., seven times over gives only 128. This excess of 1·75 is rather more than $\frac{1}{5}$ of a semitone. (To be exact, it is the very small fraction $\frac{699}{715}$ greater than $\frac{1}{5}$ of a semitone.)

Since, therefore, we may call the excess of 12 Fifths over 7 octaves about $\frac{1}{5}$ of a semitone, as this little calculation proves, then the amount to be taken from each Fifth must be about $\frac{1}{12}$ of $\frac{1}{5}$ (i.e. $\frac{1}{60}$ of a semitone), in order to reduce 12 Fifths to exactly 7 octaves.

HISTORY OF THE TUNING FORK
(PITCH).

In the year 1711, John Shore, the most famous member of a family of great trumpeters, himself Royal Trumpeter to Queen Anne—and what is of greater importance, trumpeter to Henry Purcell, perhaps the greatest of English musicians (and, as I maintain, the composer of our National Anthem, amongst other immortal masterpieces)—hit upon the device of bending a little rod of steel into hair-pin shape, and putting a handle at the bend, so as easily to hold it and strike it ; thus obtaining a standard of pitch. A bar of metal gives a clear musical note when struck and set into vibration, and this note is quite unusually permanent ; it alters so extremely slightly with rises in temperature, with age, &c., that these altera-tions may be disregarded, and John Shore's device may be accepted as giving us a nearly perfect standard pitch. (We tuners must never forget him, the father of our art : and it is pleasant to remem-ber that not only Purcell wrote many fine trumpet parts for John Shore, but that Handel, who came to England in 1712, wrote all his noble trumpet parts— for instance, the very finest of all, the trumpet obbligato to "Let the Bright Seraphim" — for this unrivalled artist. George I., who ascended the throne in 1714, kept him on as Court

Trumpeter, and also added a post for him in the Chapel Royal.)

Now the pitch of a' (at that time always used for the starting-point of tuning) was, in John Shore's day, 419·9 vibrations per second. We have the very fork that John Shore made, and that is its pitch as carefully measured by Mr. Ellis. In 1885 this fork belonged to the Rev. G. T. Driffield and was in his possession; I hope it is so still. At all events it is sure to be preserved somewhere. Since 1711 the pitch has steadily risen. Handel's fork, dated 1741, also belonging to the Rev. G. T. Driffield, gives $a' = 422\cdot5$, showing a little rise in the forty years, of $2\frac{1}{2}$ vibrations per second.

The great composer Weber, when conducting the Opera at Dresden in the year of the Battle of Waterloo (1815), used an a' of 423·2. So that from 1711 to 1815 the pitch of a' had risen 3·3 ($3\frac{3}{10}$) vibrations per second. That is only a small rise, for a whole century. But the greater use of brass instruments, always sharpening as they grow warm, now began rapidly to force up the band pitch : and in 1826 Weber's successor (Reissiger) used an a' fork of 435 per second at the Dresden Opera, a rise from Weber to Reissiger in ten or twelve years of no less than 11·8 vibrations. By 1836 the Paris Opera had sharpened to 441 (Meyerbeer), and Dresden the same (Reissiger) : and in 1879 the Covent Garden Opera pitch was $a' = 445$. At the "Music and Inventions" International Exhibition in 1885 the official pitch was 452, and Kneller Hall pitch is 452·9, practically 453. Kneller Hall, near Twickenham (the training school for military bands), is in fact the only great stumbling-block to a uniform pitch at the present day. Military pig-headedness refuses to budge a vibration : and so long as the "brass" of the orchestra depends upon soldier-musicians, this abominably high pitch cannot be everywhere lowered. But vocal-

ists began to cry out because of the strain on their voices. Every note was, as you see, $1\frac{1}{2}$ semitone above the pitch used by Purcell and Handel and Mozart. The pitch of orchestras always rises as the brass instruments grow warmer during the progress of the performance, and the fiddlers tune up to them. So that when the Vienna Opera fork reached 456, and the orchestra of course tuned to this at starting, Nacke, a competent observer, measured an a' of much over 460 (he said indeed 466) as actually played and sung at the end of the Opera in Vienna; that is considerably over $1\frac{3}{4}$ semitones above Mozart's pitch. It was time to stop. Therefore in 1859 the Paris Conservatoire with great care fixed a "Diapason Normal," a "Concert Pitch" as we should call it. And they took the Dresden fork of Reissiger (referred to above as used in 1826) and solemnly proclaimed French Pitch (July 1st, 1860) as a' 435. Mr. Ellis and the great expert Kœnig measured the actual fork, preserved under glass at the Conservatoire, and found it nearly half a vibration per second too sharp. It is really 435·4. But what matters the error of any single fork, even if it is the Standard Fork, provided we are agreed that $a' = 435$ vibrations per second?

The c'' corresponding to this a' is 522, tuned by just intonation as a Minor Third above (ratio 5 : 6), and this is now generally accepted as the Standard English Concert Pitch. Our forks are 520, giving an intentional error of 2 beats flat,—simply because it is easy to tune precisely 20 beats sharp in 10 seconds (and so to reach precisely 5220 from the fork's 5200), whereas it is impossible with all the care in the world to tune an exact unison by ear. One beat in 10 seconds is indistinguishable by nearly everyone; and there is no human ear that could hear one beat in 20 seconds. And yet such errors at Pitch c'', say respectively 5221 and 5220·5

in 10 seconds, are quite sufficient to vitiate the
unison with 5220.   Therefore in this little book all
the calculations are based upon $c' = 261$ and $c'' =
522$.

I ought in fairness to add that by Equal Tem-
perament the c'' derived from $a' = 435$ would be
rather less than 522, derived (as above) by just
intonation, — viz., 517·3.   Even then we must
remember that if a pianoforte and an orchestra
were tuned to 517·3 at the beginning of a concert
the pianoforte would be much out of tune by the
end of it, because the band would be considerably
sharper.   Therefore if the pianoforte starts at our
tuner's pitch of $c'' = 522$ and the band at $c'' = 517·3$,
the two would be very little out at the beginning
(the band being flat to the pianoforte), would be
together about half way through, and would be a
little out on the other side, at the end (the band
now being sharp to the pianoforte).   And in this
way we have halved the discrepancies ; as piano-
fortes alter very little in pitch by warmth ; indeed,
so little as not to matter at all.

I ought also to mention that in 1896 the Phil-
harmonic Society—chiefly I believe moved by this
very fact of the rise in the band pitch during a
performance, whereas there is no rise in the piano-
forte pitch—recommended an a' for pianofortes of
439 vibrations per second ; because, they said, this
would be the pitch of an orchestra which started
with $a' = 435$ (the Diapason Normal at 59° Fahr.)
and which would be at $a' = 439$ when the room
had risen to 68° Fahr.; and would even rise beyond
that, since most concert rooms would end by
being hotter than 68° at the end of the evening.
Now $a' = 439$ is $c'' = 522$ by Equal Temperament,
so on the whole I think we have done wisely
in taking $c'' = 522$ as Pianoforte Concert Pitch.

WHY THE SCALE OF A BEGINS THE ALPHABET.

You may wish me to explain a matter which must have often puzzled you, — why do not the notes of the scale of C, the principal key of all, begin A B C D, &c.? Why is this honour reserved for that queer scale made by the white notes, A B C D E F G A, which need a G♯ instead of G (and, as some would say, an F♯ too), to "make sense"? It is because this queer scale is actually one of the ancient Greek diatonic scales come down to us in a really remarkable way. The Greeks had diatonic scales, as we have, on every semitone, but they were always of this A to A pattern—a Minor Scale with a Flat Seventh, we might call it. This scale of Greek A minor was the "Hypodorian" diatonic key of ancient Greece. It was queerer as they tuned it, all by perfect Fourths and Fifths, than even it is as tuned in our equal temperament. The Greeks recognised and used the Octave as a harmony (1 : 2), but not the octave as a *scale;* their scale was only a tetrachord, or little group of four notes (the most ancient Lyres having only that compass), and there were two tetrachords in a key. These funny little tetrachord scales were always of one pattern,—viz., a semitone followed by two tones. In Greek Hypodorian (our A to A, white notes) the lower tetrachord was B C D E, where B to C is of course a semitone, C to D and D to E are tones, and the outside notes of the tetrachord from B to E are a perfect Fourth. The upper tetrachord always began on the top note of the lower, so that in this key it is E F G A ; and

here E–F is a semitone and E–A is as before a
perfect Fourth. Then from this top A, if we drop an
Octave—viz., down to a full tone below the B we
started with—we shall get what they called the
proslambanomenos, or "added tone"—i. e., added
below and beyond the two tetrachords—to fill out
the octave. With this lower *(added)* A we have
at last the complete Hypodorian key or octave
of the ancient Greeks.

Now in the break-up of ancient civilisation
and art, all music went with the rest. But as
Greek Hypodorian was the favourite diatonic
key (suiting all voices fairly, so that it was called
"The Common Key"), the organs built in the
last centuries B. C. were always tuned to it.
These organs lasted on ; and they, or their succes-
sors, thus wonderfully preserved this one key (out
of the Greek series of twelve) right into the Dark
Ages. The sacred seven notes were all the music
there was to the monks of the early centuries :
they knew no more, and thought there could be
no more in Nature. Therefore, when they thought
of using letters to distinguish musical notes, they
named this wondrous ancient scale or sequence, so
miraculously saved to them out of the world's
eclipse, with the first seven letters.

NOTE TO THE DIAGRAM ON p. 36.

Although it was mentioned on p. 33 that the
actual distance travelled by the particles of the
air in sound was "very short," the diagram on
p. 36 might be misunderstood as showing a dis-
tance of actual movement (the space *a*), which
would be an important fraction of the whole
wave (*a* to *b*). It might be wiser, therefore, to

remind the reader that a diagram is not a representation of fact, and in practice is usually exaggerated for the sake of clearness.

The amplitude — or to-and-fro swing of the actual particles of moving air in the sound wave —is usually insignificant as compared with the length of the wave itself; at all events, by the time the wave reaches your ear. Many measurements have been taken. One was taken in my own presence at the Royal Institution on May 27th, 1919, by Professor Bragg (Quain Professor in the University of London), and I may be permitted to give an account of it. A thin metal sheet was made to vibrate swiftly and steadily by electrical means, and the vibrations were counted and measured. The note f' was produced,—that is, a note with a wave 3ft. long; there were 348 vibrations (to-and-fro) a second, and the extent (amplitude) of the vibration of the plate (and therefore of the air immediately adjacent to the plate) was 7200 to the inch. Now sound decreases very rapidly in force (that means in amplitude; as it is the amplitude of a wave, the actual air movement, which gives the force), although the length of the wave always remains the same; and while at the source the amplitude was $\frac{1}{7200}$ of an inch, it would be reduced to merely some multiple of the width of a molecule at my distance (say thirty feet) from the lecture-table; that is to say, to a movement of the air particles so small as to be unrepresentable, and indeed thinkable only with difficulty.

Various attempts to give some idea of the size of a molecule have been made. We are told that if every second, day and night, a million molecules of water (and air molecules are of the same size for all we know to the contrary) had been pouring into a tumbler from the days of Adam till now, the tumbler would not be nearly full yet. For philosophers differ between four million and forty

million years as the time it would take to get half a pint of water, by additions of a million molecules a second; but no one puts it at less than four million years.

When we speak of molecular distances, we are therefore speaking of something almost inconceivably small; and the movements of air particles in most of the sound waves which reach our ear are of this character.

Waves with a large amplitude (i. e., very loud), such as those which beat upon our ears when we stand in a belfry while the bells are ringing, are actually painful to listen to, and may, I believe, cause injury to the ear. Moreover, it is impossible at such a small distance clearly to perceive the pitch of so tremendous a wave; it is more like a blow than a sound.

In fact, in everything else as well as in sound, moderation is most effective as well as most pleasant : so lest this little manual grow to immoderate length (or depth) we will bring it now to an end. If it prove interesting—and, above all, if it is of some use—to his brother tuners, the author will be well rewarded.